Cryptanalysis of DRegZ Scheme

Andrew Lamoureux

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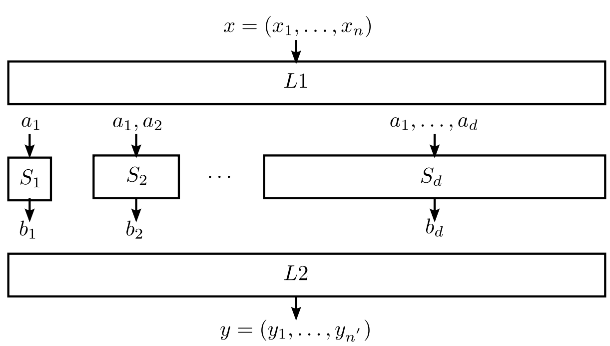
Abstract

In [1] Patarin builds his description of 2R schemes by first describing 1R schemes and their weaknesses. In [2], Bertoletti introduces DRegZ, a 1R scheme modified to counter these weaknesses. The modifications are an increased s-box size (from a suggested 8 bits to 16 bits) and an overlapping s-box structure where the input to an earlier s-box is also an input to every s-box that follows. This is intended to make the "separation of branches" from [1] more difficult. This document shows how it still remains possible to choose inputs that affect particular s-boxes, and how to these choices to decrypt ciphertext without the private data.

1 DRegZ

Reader is assumed to have read [1] and [2]. The entire function $f()$ maps n bits to n' bits. Let $a = L1(x)$, be partitioned into d groups $a_1, \ldots, a_d$ to enter the d s-boxes S_i. Denote the input size of S_i as $|S_i|$ which matches the bit width of a_i. Let b be similarly partitioned into d groups $b_1, \ldots, b_d$, the outputs of the S_i. Denote the output size of S_i as $|S_i|'$ which matches $width(b_i)$. To increase the chances a solution exists, $|S_i| > |S_i|'$. $\sum |S_i| = \sum width(a_i) = n$ and $\sum |S_i|' = \sum width(b_i) = n'$.



Attacking DRegZ occurs in two phases. First, a method is developed to carefully choose x so that the resulting $a = L1(x)$ affects only certain S_i. Second, we derive how these x are used to produce b such that $y = L2(b)$ for chosen y.

2 Isolating Inputs to Chosen S_i

Normally with many random inputs r_i, $L1(r_i)$ produces vectors a which span the entire n-dimensional space. Let s_i be the inputs where $L1(s_i) = a = (0, a_2, \ldots, a_n)$. The $a_1 = 0$ affects the dimension of a, b, and y in a way that we will exploit.

With $width(a_1) = |S_1|$ bits held constant, a now spans a subspace only $1/2^{|S_1|}$ the size of the entire n-dimensional space. $dim(a) = n - |S_1|$. Since L1 is an invertible linear transform, there exists a counterpart subspace in x.

Now b_1 is held constant, at whatever the value S_1 is evaluated at $a_1 = 0$. With $width(b_1) = |S_1|'$ bits held constant, b now spans a subspace only $1/2^{|S_1|'}$ the size of the entire n'-dimensional space. $dim(b) = n' - |S_1|'$. Since L2 is an invertible linear transform, there exists a counterpart subspace in y.

But how do we find these s_i? For the random inputs r_i already mentioned, likely their $a_i \neq 0$. Adding s_i, $f(r_i + s_i)$ should differ from $f(r_i)$ in $n' - |S_i|'$ dimensions. This is the differential attack given in [1]. To test if, for some trial vector t, $L1(t) = (0, a_2, \ldots, a_n)$, we do:

1. for each of n' random inputs r_i, collect $f(r_i) - f(r_i + t)$ into a basis B.
2. return true if $dim(B) = n' - |S_1|$, false otherwise

Now we wish to find s_i such that $f(s_i) = (0, 0, a_3, \ldots, a_n)$. Since this is a subspace of $(0, a_2, \ldots, a_n)$, we may apply the above algorithm, except the random r_i are taken from the span of $(0, a_2, \ldots, a_n)$ instead of the entire n-space. Iterating this way, we collect d bases:

$$\begin{aligned} B_1 = \{x\}|L1(x) &= (a_1, a_2, \ldots, a_d) \\ B_2 = \{x\}|L1(x) &= (0, a_2, \ldots, a_d) \\ &\ldots \\ B_d = \{x\}|L1(x) &= (0, \ldots, 0, a_d) \end{aligned}$$

Now $dim(B_1) = n, dim(B_2) = n - |S_1|, \ldots, dim(B_d) = n - \sum |S_{i,i<d}|$. For any $v \in span(B_i)$, $L1(v) = (0, \ldots, 0, a_i, a_{i+1}, \ldots, a_n)$.

Since $B_d \subset B_{d-1} \subset B_{d-2} \subset \ldots \subset B_1$, the bases can be further refined by subtracting away the subset relationships. We let B_d stand independent, now $B_{d-1} = B_{d-1} - B_d$. Next $B_{d-2} = B_{d-2} - Bd - 1$, and so on until $B_1 = B_1 - B_2$. A simple algorithm can be used to calculate the difference between two bases $B \subset A$:

1. $result = \{\}$
2. $temp = B$
3. $r = random(span(A))$

4. if $r \notin span(temp)$
   $result = result \cup \{r\}$
5. $temp = temp \cup \{r\}$ (r possibly dependent, and not increase $dim(temp)$)
6. if($dim(A) = dim(B) \neq dim(result)$)
   goto step 2
7. return $result$

Now $dim(B_1) = |S_1|, \ldots, dim(B_d) = |S_d|$. It is tempting to think that B_i needs to span vectors v where $L1(v) = a$ where $a_{j,j\neq i} = 0$, which is not true for this construction. In reality, each B_i needs only to span vectors v where $L1(v) = a$ that meet the following requirements: Each $a_{j,j<i}$ must equal 0. Each $a_{j,j=i}$ be completely controllable by choice of v. And each $a_{j,j>i}$ can be random bits dependent on $a_{j,j\leq i}$. This is because the choice $v_1 \in span(B_1)$ can be made first, dictating the value of $b_1 = S_1(a_1)$. Yes, this sets all $a_{i,i>1}$, $S_{i,i>1}$, and $b_{i,i>1}$ randomly, but only temporarily. Next, choose a vector $v_2 \in span(B_2)$ such that $b_2 = S_2(b_2)$ is set correctly (S_1 remains correct because the a_1 resulting from $L1(v_2)$ is 0). Iterate this way until S_d is set correctly.

3 Adapting Inputs Towards a Plaintext y

The last section left with a method to set S_i to our choosing. But b still remains to travel through $L2$.

Recall that evaluating $f(v), v \in span(B_i)$ produces b vectors that naturally form a space. Since $f(v), v \in B_i$ produces $a = (0, \ldots, 0, a_i, a_{i+1}, \ldots, a_d)$, all $b_{j,j<i}$ are held constant. This means the produced vector b form a subspace of the entire n'-space. As an invertible linear transformation, the values $y = L2(b)$ form also a subspace. For every B'_i, we want to record what subspace at y can be produced. Calculate each $B'_i = span(\{f(v)\})$ where $v \in span(B_i)$.

We now have a correspondance between vector spaces at the input (B_i) and vector spaces at the output (B_i'). Since $\sum dim(B'_i) = n'$ we know that the vector spaces at output can be used together to craft any output y allowable by the S_i. Decomposing some y into components from each vector space B'_i can be done by collecting the vectors from each B'_i into a matrix, preserving their order, and solving:

$$[c_1, c_2, \ldots, c_{n'}] \begin{bmatrix} v_1 \in B'_1 \\ v_2 \in B'_1 \\ \cdots \\ v_{dim(B'_1)} \in B'_1 \\ \cdots \\ v_{n'-dim(B'_d)} \in B'_d \\ v_{n'-dim(B'_d)+1} \in B'_d \\ \cdots \\ v_{n'} \in B'_d \end{bmatrix} = [y]$$

With c found, y's component from B_i is calculated as the dot product of c with the same matrix as above, except the rows pertaining to vectors from $B_{j,j\neq i}$ are set 0:

$$component_{B'_i} = \begin{bmatrix} c_1 \\ c_2 \\ \dots \\ c_{n'} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \dots \\ 0 \\ v_{\sum_{j=1}^{i-1} dim(B'_j)} \in B'_i \\ v_{\sum_{j=1}^{i-1} dim(B'_j)+1} \in B'_i \\ \dots \\ v_{\sum_{j=1}^{i} dim(B'_j)-1} \in B'_i \\ 0 \\ \dots \\ 0 \end{bmatrix}$$

We can now decompose any y into the d spaces B'_i. We have also the correspondence between these B'_i and the B_i at input. Thus we are equipped to adapt an input that solves for y with the following algorithm:

1. decompose y into the d components from each B'_i
2. find $solution_1 \in span(B_1)$ such that $component_1(f(solution_1)) = component_1(y)$
3. find $solution_2 \in span(B_2)$ such that $component_2(f(solution_1 + solution_2) = component_2(y)$
4. $\dots$
5. find $solution_d \in span(B_d)$ such that $component_d(f(solution_1+, \dots, +solution_d) = component_d(y))$
6. return $\sum solution_i$

References

[1] Jacques Patarin, Asymmetric Cryptography with S-Boxes, pp. 1-10.
[2] Giuliano Bertoletti, Algorithm for license codes (sci.crypt)